An account of a Book. *Methodus* Figurarum *lineis rectis & curvis comprehenſarum quadraturas determinandi Authore* J. Craige. 4to. Londini 1685.

THE great uſe of drawing the Tangents of Curve Lines, has made the moſt famous amongſt the Modern Mathematicians endeavour to find out General Methods of finding the Tangents of Curve Lines, as may be ſeen from the ſeveral ways invented by *Des Cartes, Monſieur Fermat, Sluſius*, Dr. *Barrow*, Dr. *Wallis, Tſchurnehuys*, and *Leibnitius*; But as yet none has attempted to invert this problem generally, that is, having the Tangent to find the Curve Line whoſe tangent it is. Therefore the Author of this Treatiſe perceiving that the doing of this would give a General Method of determinating the Quadrature of any Curvilinear ſpace, has laid down a rule for inverting *Sluſius* his method mentioned in the *Philoſophick Tranſactions Num.* 90. He has illuſtrated his Method of Quadratures by ſeveral Figures which have been already conſidered by Geometers. As for the Circle & Hyperbola, he aſſerts that their indefinite Quadratures are impoſſible, and therfore in theſe & ſuch like caſes, he expreſſes the Area by an infinite Series, which is eaſily done by his Method, except the Series conſiſt of irrational termes, for in theſe he has recourſe to *Leibnitius* his method of finding Tangents, where the Calculation will be more tedious. By his reſolving the Area of the Hyperbola into an infinite ſeries, he comes to the ſame expreſſion with that of *N. Mercator*: And in meaſuring the Zone of a Circle, his expreſſion falls in with that invented by Mr. *Iſaac Newton*, as Mr. *David Gregory* relates in his Treatise. He has ſubjoyned a Method of meaſuring the Curve Superficies made by the rotation of any Curve upon its *Axis*; with a ſmall Animadverſion on the Method of *Quadratrices*, publiſhed in the *Acta Lipſienſia Eruditorum* of *October*, 168[illegible].

Since the Publication of this Treatiſe, the Authour is pleaſed to make the following Addition.

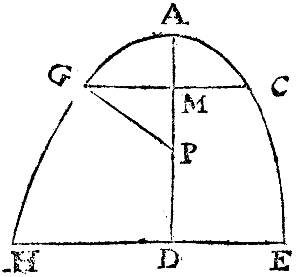
Additio ad Methodum Figurarum Quadraturas Determinandi. Autore *Johanne Craige.*

Quoniam omnium Figurarum Quadraturæ ex perfecta nostri primi problematis Solutione determinantur; propterea utile judicabam nonnulla addere, quæ solutionem meam non modo plenius illustrant sed omnino perficiunt: non adeo tamen sunt obscura, quin facile quisquam in istiusmodi rebus versatus, ex iis quæ jam exposui, omnia supplere possit. Problema sic se habet. Data expressione Analytica lineæ inter ordinatam & Curvæ perpendicularem designatæ, Invenire æquationem naturam illius Curvæ definientem. Hoc problema tres casus includit. 1. *Cum expressio istius lineæ talis est, qualis a vulgaribus tangentium Methodis exhibetur.* 2. *Cum ad simpliciorem reducitur, facta divisione numeratoris & denominatoris per communem simplicem divisorem.* 3. *Cum expressio sit simplicissima, dividendo per divisorem compositum. Duos priores casus Regula, prout eam explicui, universaliter comprehendit; superest tantum ut ostendam quo pacto tertium pariter casum comprehendat.*

Postquam expressio data per y *multiplicatur, apponantur omnes termini qui sub maximo continentur (Terminorum magnitudinem e dimensionibus quantitatis* y *mensurans) & connectantur signo affirmativo vel negativo, prout libuerit; adæquentur omnes illi termini (prius in coefficientes incognitas multiplicati) Quadrato quantitatis per* x *designatæ: eritq; inde resultans æquatio quæsita, vel quæsitam includet; & determinationes coefficientium terminos æquationem constituentes a reliquis distinguent.*

Sit

Sit in apposito schemate abscissa AM$=y$, *ordinata* MC$=z$, *& Curvæ* ACE *proprietas* $z^2=\frac{a^2y^2+y^4}{p^2}$, *& invenienda sit quadratura Areæ a lineis rectis & illa Curva comprehensæ. Quærenda est alia Curva* AGH, *in qua* PM$=\sqrt{\frac{a^2y^2+y^4}{p^2}}=z$; *ubi* PG *Curvæ quæsitæ perpendicularem, &* MG$=x$ *illius ordinatam denotat. Cumq; hæc expressio lineæ* PM *in* y *multiplicata contineat sextam quantitatis* y *dimensionem, ideo appono omnes terminos sub illa sexta dimensione contentos, unde resultans æquatio est.*



$$\frac{na^6+ma^5y+la^4y^2+ha^3y^3+ka^2y^4+gay^5+fy^6}{p^2}=x^4.$$

Ex hac æquatione invenio valorem Lineæ PM. *quem comparo cum valore dato, unde*

$$PM=\frac{ma^5+2la^4y+3ha^3y^2+4ka^2y^3+5gay^4+6fy^5}{4p\sqrt{na^6+ma^5y+la^4y^2+ha^3y^3+ka^2y^4+gay^5+fy^6}}=\sqrt{\frac{a^2y^2+y^4}{pp}}.$$

auferantur fractiones & Signa radicalia, & determinentur coefficientes n, m, l &c: *(ut in* prob: 2 *tractatus nostri) rejectis iis quarum determinationes absurdum involvunt: & cæteræ, in quibus nihil tale contingit, æquationem constituent. Sic in exemplo proposito, erit* $f=\frac{4}{9}$, $k=\frac{12}{9}$, $l=\frac{12}{9}$, $n=\frac{4}{9}$. *sed dum* g *determino, invenio* $240g=144g$, *quod absurdum involvit; & sic pro h comparatio erit* $16h=16h$ *unde nullus illius valor, & pro* m *erit* $4m=4m$, *quod itidem est absurdum: quapropter termini a quantitatibus* g, h, m *affecti ad æquationem non pertinebunt; unde reliqui a literis* n, l, k, f *affecti æquationem naturam Curvæ definientem constituent. sc.*

$$\frac{4a^6+12a^4y^2+12a^2y^4+4y^6}{9p^2}=x^4,$$

$=x^4$, *adeoq;* AMC$=\sqrt{\frac{a^6+3a^4y^2+3a^2y^4+y^6}{6p^2}}=\frac{x^2}{2}$. *Exem.* 2

Sit Curvæ Lineæ ACE *talis proprietas* $z^2=\frac{qy^2+y^3}{q}$ *& inveniendæ sit Quadratura spatii* AMC. *Quærenda est Curva* AGH *in qua sit* PM$=\sqrt{\frac{qy^2+y^3}{p}}=Z$ *& quoniam hic valor in* y *multiplicatus continet quintam quantitatis* y *dimensionem, apponantur omnes termini sub illa quinta dimensione, & æquentur quadrato quantitatis per* x *designatæ; unde æquatio resultans est.*

$\frac{nq^5+mq^4y+lq^3y^2+kq^2y^3+hqy^4+fy^5}{p}=x^4$. *atq; sola coefficientis* (m) *determinatio absurdum involvet, eruntq; reliquæ,* $n=\frac{64}{225}$, $l=-\frac{16}{15}$, $k=-\frac{16}{45}$, $h=\frac{16}{15}$, $f=\frac{16}{25}$, *unde æquatio curvam quæsitam definiens est.*

$\frac{64q^5}{225p}-\frac{16q^3y^2}{15p}-\frac{16q^2y^3}{45p}+\frac{16qy^4}{15p}+\frac{16y^5}{25p}=x^4$ *adeoq;*

AMC$=\sqrt{\frac{16q^5}{225p}-\frac{4q^3y^2}{15p}-\frac{4q^2y^3}{45p}+\frac{4qy^4}{15p}+\frac{4y^5}{25p}}=\frac{xx}{2}$

Exem. 3. *Inuenienda sit Quadratura spatii* AMC, *definita natura Curvæ* ACE *hac Æquatione* $z^2=\frac{a^3}{4y+4a}$. *Quæratur alia Curva* AGH, *in qua* PM$=\sqrt{\frac{a^3}{4y+a}}=z$. *Ex præmissis constat Æquationem primam fore* $\frac{na^3y^2+ma^4y+16a^5}{4a+4y}=4x$ *& determinationes Coefficientium* $n=16$, $m=32$, $l=16$. *Quibus substitutis, erit æquatio* $\frac{16a^3y^2+32a^4y+16a^5}{4y+4a}=x^4$ $=4a^3y+4a^4$: *adeoq;* AMC $=\sqrt{a^3y+a^4}=\frac{1}{2}x^2$.

Notatu dignissimum est, has tres (sicut infinitas alias) Quadraturas abscissæ AM *(seu* y*) non convenire. Quoniam in istiusmodi*

modi Figuris, ſimpliciſſima Areæ expreſſio huic portioni non reſpondet: attamen Quadratura abſciſſæ conveniens exinde parvo labore deducitur. Ut in Exem: 3. *ubi Area eſt* $\sqrt{a3y+a4}$; *fiat* y $=0$, *&* *erit Area* $\sqrt{a4}=a2$, *& ſubducatur hæc ex generali, proveniet Quadratura portionis abſciſſæ reſpondentis, ſc.* $\sqrt{a3y+a4}-a2$. *Quam obſervatiunculam mihi primus ſignificavit Vir celeberrimus D. Iſaacus Newton.*

Tentetur jam idem proceſſus in Circulo ACE, *cujus diameter ſit* r, *ac proinde* $Z=\sqrt{ry-y2}$, *Quærenda eſt Curva* AGH *in qua* $PM=\sqrt{ry-y2}=z$, *ſed ex dictis conſtat æquationem primam fore* $nr4+mr3y+fr2y2+hry3-ky4=x4$: *& ſingulæ coefficientium determinationes erunt impoſſibiles; adeoq; nulla datur Curva* AGH *in qua* $PM=\sqrt{ry-y2}$, *ac proinde Circuli Quadratura indefinita eſt impoſſibilis. Fieri tamen poteſt ut ſit aliqua hujuſmodi Curva* AGH, *ſed ex earum numero, quas poſt* Carteſium *Mechanicas Geometræ communiter appellant: ſed quia harum uſus non libenter admittunt Mathematici, præſtat hujuſmodi Quadraturas per ſeries infinitas exhibere.*

Benevole Lector

Ob inopiam Typorum Numeralium minuſculorum, qui ad deſignandas quantitatum poteſtates ſupra Symbola dextrorſum apponi ſolent, feſtinante prælo, Typographus paulo majoribus uſus eſt in eadem linea immediate ſequentibus; ubicunq; itaq; offenderis a3, *vel* x2, *&c. cubum vel quadratum, &c.* e *quantitate, cui ſuffigitur numerus, intelligas.*

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